Punching shear stress check and reinforcement design based on ACI code

Categorization of columns:

Based on the geometry of the floor slab at the vicinity of a column, each column is categorized into one of the following options:

1. Interior column
   Each face of the column is at least four times the slab thickness away from a slab edge

2. Edge column
   One side of the column normal to the axis of the moment is less than four times the slab thickness away from the slab edge

3. Corner column
   Two adjacent sides of the column are less than four times the slab thickness from slab edges parallel to each

4. End column
   One side of the column parallel to the axis of the moment is less than four times the slab thickness from a slab edge

In cases 2, 3 and 4, column is assumed to be at the edge of the slab. The overhang of the slab beyond the face of the column is not included in the calculations. Hence, the analysis performed is somewhat conservative.

Stress calculation:

Stress is calculated for several critical perimeters around the columns based on the combination of the direct shear and moment:

\[ V_u = \frac{V_u}{A} + \frac{\gamma \times M_u \times c}{I} \]

where \( V_u \) is the absolute value of the direct shear and \( M_u \) is the absolute value of the unbalanced column moment about the center of geometry of the critical section. \( c \) is the distance of the point of interest to the center of the critical section, \( A \) is the area of the critical section, \( \gamma \) is the ratio of the moment transferred by shear and \( I \) is the moment of inertia of the critical section about the axis of moment.

The implementation of the above in ADAPT is provided with the option of allowing the user to consider the contribution of the moments separately or combined. ACI 318
however recommends that due to the empirical nature of its formula, punching shear check should be performed independently for moments about each of the principal axis.

For a critical section with dimension of $b_1$ and $b_2$ and average depth of $d$, $A$, $I$, $c$, $\gamma$ and $M_u$ are:

1. Interior column:

   $A = 2(b_1 + b_2)d$
   $c = \frac{b_1}{2}$
   $I = \frac{b_1 d^3}{6} + \frac{db_1^3}{6} + \frac{b_1^2 b_2 d}{2}$
   $\gamma = 1 - \frac{1}{1 + \frac{2}{3} \frac{b_1}{b_2}}$
   $M_u = \text{abs}[M_{u,\text{direct}}]$

2. End column: ($b_1$ is perpendicular to the axis of moment)

   $A = (2b_1 + b_2)d$
   $c = \frac{b_1^2 d}{2b_1 d + b_2 d}$
   $I = \frac{b_1 d^3}{6} + \frac{db_1^3}{6} + 2b_1 d \left( \frac{b_1}{2} - c \right)^2 + b_2 d c^2$
   $\gamma = 1 - \frac{1}{1 + \frac{2}{3} \frac{b_1}{b_2}}$
   $M_u = \text{abs}[M_{u,\text{direct}} - V_u \left( \frac{b_1}{2} - c \right)]$

3. Corner Column:

   $A = (b_1 + b_2)d$
   $c = \frac{b_1^2}{2b_1 + 2b_2}$
   $I = \frac{b_1 d^3}{12} + \frac{db_1^3}{12} + b_1 d \left( \frac{b_1}{2} - c \right)^2 + b_2 d c^2$

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1 “Concrete Q&A- Checking Punching Shear Strength by the ACI code,” Concrete International, November 2005, pp 76.
\[
\gamma = 1 - \frac{1}{1 + \frac{2}{3} \sqrt[3]{b_2}} \\
M_u = abs\left( M_{u,\text{direct}} - V_u (\frac{b_1}{2} - c) \right)
\]

4. Edge column: \((b_1\) is perpendicular to the axis of moment)

\[
A = (b_1 + 2b_2)d \\
c = \frac{b_1}{2} \\
I = \frac{b_1 d^3}{12} + \frac{db_2^3}{12} + 2b_2 dc^2 \\
\gamma = 1 - \frac{1}{1 + \frac{2}{3} \sqrt[3]{b_2}} \\
M_u = abs\left[ M_{u,\text{direct}} \right]
\]

**Allowable stress:**

Allowable stress of the section is calculated based on the following equation:

\[
v_c = \beta_p \sqrt{f'_c} + 0.3 f_c
\]

where \(f_c\) is the average stress due to post tensioning, \(f'_c\) is the strength of the concrete and \(\beta_p\) is the smaller of 3.5 and \((1.5 + \alpha_s d/u)\). \(\alpha_s\) is 40 for interior columns, 30 for edge and end columns and 20 for corner columns. \(u\) is the perimeter of the critical section.

If any portion of the column cross section closer to a discontinuous edge than four times the slab thickness or \(f'_c\) is less than 125 psi, then the allowable stress is calculated based on:

\[
v_c = \min \left\{ \begin{array}{l}
(2 + \frac{4}{\beta_c}) \sqrt{f'_c} \\
(2 + \alpha_s \frac{d}{u}) \sqrt{f'_c} \\
4 \sqrt{f'_c}
\end{array} \right. 
\]

where \(\beta_c\) is the ratio of the larger to the smaller side of the critical section.
Critical sections

The closest critical section to check the stresses is \( d/2 \) from the face of the column where \( d \) is the effective depth of the slab/drop cap. Subsequent sections are 0.5\( d \) away from the previous critical section.

If drop cap exists, stresses are also checked at 0.5\( d \) from the face of the drop cap in which \( d \) is the effective depth of the slab. Subsequent sections are 0.5\( d \) away from the previous critical section.

Stress check:

Calculated stresses are compared against the allowable stress:

\[
\text{If } v_u < \varphi v c \quad \text{no punching shear reinforcement is required}
\]

\[
\text{If } v_u > \varphi_n v_{\text{max}} = \varphi_v^* 6\sqrt{f'_c} \quad \text{punching stress is excessive; revise the section}
\]

\[
\text{If } \varphi_n v_{\text{max}} > v_u > \varphi v c \quad \text{provide punching shear reinforcement}
\]

Where \( \varphi_v \) is the shear factor and \( v_{\text{max}} \) is the maximum shear stress that can be carried out by the critical section including the stresses in shear reinforcement.

For the option of considering moments separately, if stress is below the permissible value in both directions, then no shear reinforcement is needed otherwise if at least in one direction, stress exceeds the permissible value, shear reinforcement should be provided.

Stress check is performed until no shear reinforcement is needed anymore. In case of existence of a drop cap, stresses are checked within the drop cap until the stress is less than the permissible stress and then checked outside the drop cap region until the stress is less than the permissible value.

Shear reinforcement:

Where needed, shear reinforcement is provided according to the following:

\[
A_s = \left( v_u - v_c \right) \frac{u s}{\varphi_v f_y \sin(\alpha)}
\]

where \( \alpha \) is the angle of shear reinforcement with the plane of slab and \( u \) is the periphery of the critical section. \( s \) is the spacing between the critical sections \( [d/2] \).
Arrangement of shear reinforcements:

Shear reinforcement can be in the form of shear studs or shear stirrups (links). In case of shear links, the number of shear links ($N_{\text{shear\_links}}$) in a critical section and distance between the links ($\text{Dist}_{\text{shear\_links}}$) are given by:

\[
N_{\text{shear\_links}} = \frac{A_s}{A_{\text{shear\_link}}},
\]

\[
\text{Dist}_{\text{shear\_links}} = \frac{u}{N_{\text{shear\_links}}}.
\]

If shear studs are used, the number of shear studs per rail ($N_{\text{shear\_studs}}$) and the distance between the studs ($\text{Dist}_{\text{shear\_studs}}$) are given by:

\[
N_{\text{shear\_studs}} = \frac{A_s}{A_{\text{shear\_stud}} \times N_{\text{rails}}},
\]

\[
\text{Dist}_{\text{shear\_studs}} = \frac{d/2_{\text{slab}}}{N_{\text{shear\_studs}}}.
\]