SHORTENING CALCULATION OF POST-TENSIONED FLOOR SYSTEMS

First draft, May 1, 2007

Bijan O Aalami

This Technical Note outlines the factors that cause shortening of post-tensioned floor systems. It provides guidelines for estimating anticipated long- and short-term shortenings. The concepts and procedures are illustrated through several numerical examples. A companion technical note describes the means for mitigating the adverse effect of the supports’ restraints on shortening of post-tensioned floors.

The presentation of the material is organized as follows:

1 - FACTORS CAUSING SHORTENING
2 - REPRESENTATIVE FLOOR SYSTEM
3 - ESTIMATE OF LONG-TERM SHORTENING
   Shrinkage
   Creep
   Elastic Shortening
   Temperature
4 - EXAMPLES
   Shortening calculation of a slab
5 – ESTIMATE OF SHORT-TERM SHORTENING
6 – NOTATIONS
7 - REFERENCES

1 - FACTORS CAUSING SHORTENING

The total shortening of a post-tensioned member can be expresses as follows:

\[ a = L \times (ES + SH + CR + TEM) \]

Where,

- \( a \) = total shortening;
- \( L \) = length of the member; and
- \( ES \) = elastic shortening strain;
- \( SH \) = shrinkage shortening strain; and
- \( CR \) = creep shortening strain;
- \( TEM \) = strain due to drop in temperature.

\[ \text{Shrinkage} \]
\[ \text{Creep} \]
\[ \text{Elastic Shortening} \]
\[ \text{Temperature} \]

\[ \text{Shortening calculation of a slab} \]

\[ \text{Estimate of short-term shortening} \]

\[ \text{Notations} \]

\[ \text{References} \]

---

1 Copyright ADAPT Corporation 2007
2 Professor Emeritus, San Francisco State University
For pre-defined environmental and stress conditions, referred to as standard condition, creep and shrinkage values are determined through extensive laboratory tests. The values obtained are referred to as “base creep” and “base shrinkage” strains.

The base-shrinkage strain reflects the total reduction in length over the original length of a specimen of concrete if the concrete specimen is allowed to freely shorten over an infinite length of time, under a constant pre-defined environmental condition. Likewise, the base creep strain is the total strain of a concrete specimen per unit stress over an infinite length of time, when loaded at a given age and allowed to shorten under controlled environmental conditions.

Real-life structures do not match the environment of the test specimens. To estimate the shortening of real-life structures, the practice is to start with the base creep and shrinkage values of a standard lab specimen and adjust the base values to reflect the conditions of the actual structure. The adjustment is done by applying various correction factors, each of which accounts for one of the variations between the actual structure and the standard test. As an alternative, some building codes provide recommendations to estimate the base values that are otherwise obtained from lab specimens. A third alternative is to use a test specimen from the concrete of the structure being designed, and observe its creep and shrinkage response over a short period of time (30 to 50 days) in the same environment of the actual structure. Code-recommended relationships can then be used to extrapolate the observed values to the long term base strains.

This Technical Note offers guidelines for the selection of the base shrinkage and creep values, and the corrections needed to make them applicable to real structures. The information provided is based on the recommendations of ACI [209, 1992]; AASHTO [1994], [Collins and Mitchell. 1991] and the European Code (EC2) [2004]. The information in the literature is supplemented by more than 25 years of experience in design and observation in performance of post-tensioned buildings, as well as attention to standard of practice. In order to enhance the practical value of this Technical Note to design engineers, the material presented concentrates on the conditions that are generally met in design of post-tensioned floor systems. For other conditions, apart from the preceding references, there is a vast amount of literature on factors affecting the volume change in concrete. As a result, the application of this Technical Note is limited to constructions that fall within the range of parameters detailed next under “Representative Floor System.”

2 - REPRESENTATIVE FLOOR SYSTEM

The parameters that define the application of the majority of post-tensioned floor systems are:

*Structural System:* two-way flat plates, floors with slab bands, drop caps/panels, waffle and joist construction as well as beam and slab structures, single and flanged beams.

*Post-Tensioning System:* Either unbonded, or bonded, mono- or multi-strand tendons

Concrete weight: \( W = 140 - 155 \text{ pcf} \quad (2300 - 2600 \text{ kg/m}^3) \)
Concrete strength (28 day cylinder) \( f'_c = 3000 \text{ to } 6000 \text{ psi} \quad (21 \text{ to } 40 \text{ MPa}) \)
Average precompression \( P/A = 100 \text{ to } 350 \text{ psi} \quad (0.8 \text{ to } 2.40 \text{ MPa}) \)

3 - ESTIMATE OF SHORTENING

3.1 Shrinkage

In the absence of laboratory tests, or code recommended calculations, assume the following:
Base-shrinkage strain ($SH_0$) = 500 to 600 micro-strain$^3$ ($x10^6$) for water to cement ratio 4 to 4.5

The base-shrinkage strain commonly used for structures in the San Francisco Bay Area is around 500 micro-strain. Due to better quality of aggregate, in many other parts of California the base strain for shrinkage is somewhat lower. In the absence of better information, for other areas in the US use 550 micro-strain.

For structures in other regions of the world, where local information is not available use 650 micro-strain for a first conservative estimate$^4$.

Modify the base-shrinkage strain by the size effect of the member ($k_{v/s}$ for volume-to-surface ratio $V/S$), and the ambient relative humidity of the structure ($k_{RH}$).

$$SH = SH_0 * k_{RH} * k_{v/s}$$

Adjust the base-shrinkage strain by the coefficient $k_{RH}$ given in the following table.

<table>
<thead>
<tr>
<th>Relative Humidity</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{RH}$</td>
<td>1.43</td>
<td>1.29</td>
<td>1.14</td>
<td>1.00</td>
<td>0.86</td>
<td>0.43</td>
<td>0.00</td>
</tr>
</tbody>
</table>

The base-shrinkage strain recommended was based on a volume-to-surface ratio equal to 1.5 inch (38 mm). Use the following relationships to adjust the base shrinkage for other cross-sections:

$$k_{v/s} = \frac{[1064 - 94 (V/S)]}{923} \quad \text{US units (inch)}$$

$$k_{v/s} = \frac{[1064 - 3.7 (V/S)]}{923} \quad \text{SI units (mm)}$$

The surface area used in determining the volume-to-surface area should include only the area that is exposed to atmospheric drying. For poorly ventilated enclosed cells, only 50% of the interior perimeter should be used in calculating the surface area.

**Example: 3.1-1**

Calculate the volume to surface ratio of the following sections:

(i) Slab of uniform thickness $h$

Consider a representative strip of unit width

$$V/S = \frac{h*1}{2*1} = h/2$$

Hence, for a 10" (250 mm) slab, $V/S = 5"$ (125 mm)

---

$^3$ Each micro-strain is $1x10^6$ strain

$^4$ The Hong Kong building code recommends a base shrinkage strain of 832 micro strain (Clause 4.3.4 Cs*K1)
(ii) Waffle slab with the following dimensions for each representative waffle:

- Width = 1000 mm (40")
- Depth = 500 mm (20")
- Stem width = 250 mm (10")
- Stem depth = 400 mm (16")

\[
V = 1000 \times 100 + 250 \times 400 = 200000 \text{ mm}^2
\]
\[
S = 1000 \times 2 + 400 \times 2 = 2800 \text{ mm}
\]
\[
\frac{V}{S} = \frac{200000}{2800} = 71.43 \text{ mm (2.85")}
\]

**Example 3.1-2**

For a base-shrinkage strain of 550 micro strain, what is the long-term shrinkage strain (SH) of a 250 mm slab of uniform thickness at a location with the ambient relative humidity H=80%.

Shrinkage strain, \( SH = SH_0 \times k_{RH} \times k_{v/s} \)

\[
SH_0 = 550 \times 10^{-6}
\]
\[
k_{RH} = 0.86 \quad [\text{From Table 3.1-1}]
\]
\[
\frac{V}{S} = \frac{h}{2} = \frac{250}{2} = 125 \text{ mm (5")}
\]
\[
k_{v/s} = \frac{[1064 - 3.7 \times (V/S)]/923}{\text{SI units (mm)}}
\]
\[
= \frac{[1064 - 3.7 \times 125]}{923}
\]
\[
= 0.65
\]
\[
k_{v/s} = \frac{[1064 - 94 \times (V/S)]/923}{\text{US units (inch)}}
\]
\[
= \frac{[1064 - 94 \times 5]}{923}
\]
\[
= 0.64
\]

Shrinkage strain, \( SH = 550 \times 10^{-6} \times 0.86 \times 0.65 \)

\[
= 307 \times 10^{-6}
\]

**3.2 Creep**

Creep is primarily a function of applied stress. Creep shortening of concrete under sustained load is generally in the range of 1.5 to 4.0 times the initial elastic shortening, depending mainly on concrete maturity at the time of loading. The base-creep coefficient used for post-tensioned floor systems in the US, when tendons are generally stressed in three to four days is 2.0. An upper bound value for the base-creep coefficient for the general case of post-tensioned floor systems is recommended as 2.5.

To account for the particulars of the building under consideration, the base-creep coefficient selected for a floor system must be modified as detailed in the following:

\[
CR_C = CR_0 \times K(PT) \times k_r \times k_{CRH} \times k_c
\]

Where,

- \( CR_C \) = creep coefficient;
- \( CR_0 \) = base creep coefficient;
- \( K(PT) \) = correction factor for the average precompression from post-tensioning;
\[ k_f = \text{correction factor for concrete strength}; \]
\[ k_{cRH} = \text{correction factor for the ambient humidity}; \]
\[ k_c = \text{correction factor for volume to surface ratio}. \]

The correction factor \( K(PT) \) for the range of average precompression used in building structures and the commonly used concrete strength is 1. This simplifies the calculation of shortening due to creep effects to the following:

\[ CR_C = CR_0 \times k_f \times k_{cRH} \times k_c \]

Other correction factors are:

\[ k_f = \frac{1}{0.67 + \frac{f'c}{9}} \quad \text{(US units; \( f'c \) in ksi)} \]

\[ k_f' = \frac{62}{42 + f'c} \quad \text{(SI units, \( f'c \) in MPa)} \]

\[ k_{cRH} = (1.58 - H/120) \]

Where \( H \) is the ambient relative humidity at location of the building project.

The primary impact of volume-to-surface ratio on creep shortening is over the first couple of months, when the shrinkage of concrete is more significant. The impact of volume-to-surface ratio on the long-term creep of a member is not as significant. The following relationships give the adjustment to the base creep coefficient for the member’s volume-to-surface ratio.

\[ k_c = \left[ \frac{1.80 + 1.77e^{-0.54(V/S)}}{2.587} \right] \quad \text{(US units; inches)} \]

\[ k_c = \left[ \frac{1.80 + 1.77e^{-0.021(V/S)}}{2.587} \right] \quad \text{(SI units; mm)} \]

### 3.3 Elastic Shortening

Elastic shortening is an immediate response and strictly speaking should be grouped under the section “Estimate of Short-Term Shortening.” However, since its magnitude is required for the estimate of total long-term shortening of a member, it is addressed in the following.

Elastic shortening of a member is due to average precompression over its length. The average precompression is calculated using the average force over the length of a tendon divided by the tributary of the tendon. In practice, the average force over the tributary of a design strip\(^5\) is used in the calculation.

\(^5\) Such as a beam with its entire tributary or a line of column supports with their entire tributary on each side.
Average strain due to elastic shortening is:

\[ ES = \frac{(P/A)}{E_{ci}} \]

Where,
- \( ES \) = total strain due to average elastic shortening;
- \( P \) = average value of prestressing force allowing for friction losses, but not long-term stress losses\(^6\);
- \( A \) = cross-sectional area of the member’s tributary; and
- \( E_{ci} \) = modulus of elasticity of concrete at time of stressing.

In the U.S. codes, the procedure is to determine the compressive strength of concrete at the time of stressing, and then use it to calculate the modulus of elasticity \( E_{ci} \) and thereby the shortening strain. The following are the steps:

(a) Design Based on U.S. Codes

The modulus of elasticity of concrete is given by:

\[ E_{ci} = 33 \times w_c^{1.5} \sqrt[1.5]{f'_{ci}} \]  \hspace{1cm} \text{in US units}

\( f'_{ci} \) = compressive strength of concrete cylinder at time of stressing, psi; and
\( w_c \) = weight of one cubic ft of concrete, between 90 and 155 lb/ft\(^3\); and
\( E_{ci} \) = modulus of elasticity of concrete at day of stressing, psi.

In SI units the relationship is:

\[ E_{ci} = 0.043 \times w_c^{1.5} \sqrt[1.5]{f'_{ci}} \]  \hspace{1cm} \text{in SI units}

Where, \( E_{ci} \) is in MPa; \( w_c \) in kg/m\(^3\) and \( f'_{ci} \) in MPa

If cylinder strength at time of stressing is not available, the following relationship is used to estimate its value. In most projects, however, specifications call for tendons to be stressed, when concrete reaches a cylinder strength of minimum 3000 psi (20 MPa).

\[ f'_{ci} = \left[ \frac{1.45 \times t^{0.75}}{t^{0.75} + 5.5} \right] f'_{c} \]

where \( f'_{ci} \) is concrete cylinder strength on day (t).

Using the European Code EC2, the associated relationships are given below:
Modulus of elasticity of concrete (\( E_{c} \)) at day 28 is

---

\(^6\) It is recognized that when tendons are stressed one after the other, force in previously stressed tendons will be dropped when new tendons are stressed. Since the relationship is based on the average prestressing force \( P \), no adjustment in the expression for sequence of stressing is necessary.
Modulus of elasticity on day \( t \) is given by:

\[
E_c(t) = \left( \frac{f_{cm}(t)}{f_{ck} + 8} \right)^{0.3} E_c
\]

Where:

\[
f_{cm}(t) = \exp \left( s \left[ 1 - \left( \frac{28}{t} \right)^{1/2} \right] \right) (f_{ck} + 8)
\]

\( f_{cm}(t) \) = mean compressive strength of concrete cylinder on day “t;”
\( t \) = age of concrete in days; and
\( s \) = a coefficient which depends on the type of cement, (equals 0.2 for most common cements).

### 3.4 Temperature Effects

Temperature effects are reversible depending on whether there is a rise or fall in temperature. It is generally not considered for common structures, when calculating the long-term shortening of a floor slab.

\[
d = L \cdot T \cdot \alpha
\]

Where,

\( d \) = change in length of a member;
\( T \) = change in temperature (degrees F or C); and
\( \alpha \) = coefficient of thermal expansion.

In the absence of more precise data, the coefficient of thermal expansion of concrete may be taken as:

\[
\alpha = 6.0 \times 10^{-6} /F^\circ
\]

\[
\alpha = 10.1 \times 10^{-6} /C^\circ
\]

### 4 – EXAMPLES

4.1 Estimate the long-term shortening of the following post-tensioned slab.

**GIVEN**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete</td>
<td>5000 psi</td>
<td>(34 MPa)</td>
</tr>
<tr>
<td>Slab thickness</td>
<td>8 inch</td>
<td>(200 mm)</td>
</tr>
<tr>
<td>Length of the slab</td>
<td>100 ft</td>
<td>(30 m)</td>
</tr>
<tr>
<td>Relative humidity</td>
<td>H = 75%</td>
<td></td>
</tr>
<tr>
<td>Average precompression</td>
<td>150 psi</td>
<td>(1.0 MPa)</td>
</tr>
<tr>
<td>Stressing day</td>
<td>3 day</td>
<td>(3 day)</td>
</tr>
<tr>
<td></td>
<td>but not at f'c less than 2500 psi</td>
<td></td>
</tr>
</tbody>
</table>
Change in daily temperature 25°F (14°C)

REQUIRED
Total long-term unimpeded change in length and anticipated daily change in length.

In the absence of more accurate data, make the following somewhat conservative assumptions for the base values\(^7\). These are applicable for most areas, unless quality of concrete delivered, placed and cured is known to be poor, in which case more conservative values are recommended.

- Base shrinkage strain = \(600 \times 10^{-6}\) strain
- Base creep coefficient = 2.5

Elastic shortening strain, \(ES\):

\[
ES = \frac{(P/A)}{E_{ci}}
\]

The modulus of elasticity of concrete at day 3 is expressed in terms of concrete strength on day 3 \(f'_{ci}\).

\[
f'_{ci} = \left[ \frac{(1.45 \times t^{0.75})}{(t^{0.75} + 5.5)} \right] \times f_c
\]

\[
f'_{ci} = \left[ \frac{(1.45 \times 3^{0.75})}{(3^{0.75} + 5.5)} \right] \times 5000 = 2,124 \text{ psi (14.35 MPa)} < 2500 \text{ psi}
\]

Use \(f'_{ci} = 2500 \text{ psi (17.24 MPa)}\)

\[
E_{ci} = 33 \times 150^{1.5} \sqrt{2500} = 3,031,244 \text{ psi (20,481 MPa)}
\]

In SI units the relationship is:

\[
E_{ci} = 0.043 \times 2400^{1.5} \sqrt{17.24} = 20992 \text{ MPa}
\]

Hence, the elastic shortening strain of the slab is:

\[
ES = \frac{(P/A)}{E_{ci}}
\]

\[
ES = \frac{150}{3031244} = 50 \times 10^{-6}
\]

Shrinkage shortening strain, \(SH\):

\[
SH = SH_0 \times k_{RH} \times k_{v/s}
\]

Correction for relative humidity \(H = 75\%\). From Table 3.1-1

- \(k_{RH}\) for 70\% = 1.00
- \(k_{RH}\) for 80\% = 0.86

\[
k_{RH \ 85} = 1.00 - 0.5 \times (1.00 - 0.86) = 0.93
\]

\(^7\) For structures in USA, assume base creep coefficient = 2 and base shrinkage strain = 400
Correction for volume to surface ratio:

\[ V/S = 0.5 \times 8 = 4 \text{ inch} \quad (0.5 \times 200 = 100 \text{ mm}) \]

The correction factor \( k_{v/s} \) is:

\[
k_{v/s} = \frac{[1064 - 94 \times 4]}{923} = 0.75 \quad \text{US units (inch)}
\]

\[
k_{v/s} = \frac{[1064 - 3.7 \times 100]}{923} = 0.75 \quad \text{SI units (mm)}
\]

Hence the long-term shrinkage is:

\[ SH = 600 \times 10^{-6} \times 0.93 \times 0.75 = 419 \times 10^{-6} \]

Creep-shortening strain, CR:

\[
CR = CR_c \times ES
\]

\[
CR_c = CR_0 \times k_f \times k_{CRH} \times k_c
\]

Correction for concrete strength \( k_f \) ; \( f'_c = 5000 \text{ psi} \, (34 \text{ MPa}) \)

In American units:

\[
k_f = \frac{1}{(0.67 + 5/9)} = 0.82
\]

In SI units:

\[
k_f = \frac{62}{(42 + 34)} = 0.82
\]

Correction for relative humidity

\[
k_{CRH} = (1.58 - H/120)
\]

\[
k_{CRH} = (1.58 - 75/120) = 0.96
\]

Correction for volume-to-surface ratio:

\[ V/S = 0.5 \times 8 = 4 \text{ inch} \quad (0.5 \times 200 = 100 \text{ mm}) \]

The correction factor \( k_c \) is:

In American units:

\[
k_c = \frac{(1.80 + 1.77 \times e^{-0.54\times4})}{2.587} = 0.77
\]

In SI units:

\[
k_c = \frac{(1.80 + 1.77 \times e^{-0.0213\times100})}{2.587} = 0.78
\]
Having obtained the correction factors, the creep coefficient is given by:

\[
CR_c = 2.5 \times 0.82 \times 0.96 \times 0.78 = 1.54
\]

\[
CR = CR_c \times ES = 1.54 \times 50 \times 10^{-6} = 76 \times 10^{-6}
\]

Total shortening, without taking temperature effect into account:

\[
a = L(ES + SH + CR)
\]

\[
= 100 \times 12 \times (50 + 419 + 76) \times 10^{-6}
\]

\[
= 0.65" \ (17 \text{ mm})
\]

Temperature effect:

\[
d = L \times T \times \alpha = 100 \times 12 \times 25 \times 6.0 \times 10^{-6}
\]

\[
= 0.18" \ (4.6 \text{ mm})
\]

Total shortening including temperature effect:

\[
= 0.65 + 0.18 = 0.83" \ (21 \text{ mm})
\]

5 – ESTIMATE OF SHORT-TERM SHORTENING

When designing for crack mitigation of post-tensioned floor systems due to restraint of the supports, it is necessary to estimate the short-term shortening of the floor. For the estimate of short-term shortening, the practice is to first calculate anticipated total long-term shortening of the slab. Then determine what fraction of the long-term shortening takes place over the time period required for design. Among the parameters that were described in the preceding to contribute to the long-term shortening of a floor system, shrinkage and creep are time-dependent. Their variation with time is given by the following relationships [ACI 209, AASHTO]:

5.1 Variation of shrinkage with time

The fraction of total shortening due to shrinkage that takes place on day “t” is given by:

\[
\text{Coefficient of time for shrinkage} = \frac{t}{35 + t}
\]

5.2 Variation of creep with time

The fraction of the total creep that takes place on day “t” is given by the following relationship, where “t_i” is the day of application of load (prestressing) and “t” is the day for which the fraction of total creep is sought.
Coefficient of time for creep = \( \frac{(t - t_0)^{0.6}}{10.0 + (t - t_0)^{0.6}} \times t^{0.118} \)

5.3 Short-Term Shortening Estimate Graph

For post-tensioned floor systems that fall within the representative parameters specified in this Technical Note, the combined effects of creep and shrinkage from the above relationships can be approximated through the graph shown in Fig. 5.3-1. The graph provides a first approximation to the fraction of total creep and shrinkage that takes place at a given age of concrete.

![Graph showing variation with time of the combined effects of creep and shrinkage shortening for post-tensioned floor systems](image)

**FIGURE 5.3-1**

VARIATION WITH TIME OF THE COMBINED EFFECTS OF CREEP AND SHRINKAGE SHORTENING FOR POST-TENSIONED FLOOR SYSTEMS (FELT104)

5.4 Example

The long-term shortening of a 150 ft (45.7 m) slab is estimated to be 1.25 inch (32 mm). What is the anticipated shortening of this slab on day 10 and day 28.

Referring to Fig. 5.1-1, the fraction of the shortening due to creep and shrinkage is 24% for 10 days and 43% for 28 days.

\[
\text{Shortening on day 10} = 0.24 \times 1.25 = 0.30 \text{ inch (8 mm)} \\
\text{Shortening on day 28} = 0.43 \times 1.25 = 0.54 \text{ inch (14 mm)}
\]
6 - NOTATIONS

A = cross sectional area of concrete associated with tributary of prestressing force P, in², mm²;

a = total shortening of a member, in, mm;

CR = contribution of creep strain to shortening (Non-Dimensional ND);

CR₀ = base creep coefficient, ND;

CRC = creep coefficient, ND;

d = change in length of a member;

E_c = modulus of elasticity of concrete on day 28, psi, MPa;

E_ci = modulus of elasticity of concrete on day of stressing, psi, MPa;

ES = total strain due to average elastic shortening;

f'_c = 28 day concrete cylinder strength, psi, MPa;

f'_{ci} = concrete cylinder strength on day of stressing, day t, psi, MPa;

f_{cm}(t) = mean compressive strength of concrete cylinder at age “t” days;

k_c = volume-to-surface correction factor for CR₀, ND;

k_{cRH} = correction factor for CR₀ for ambient relative humidity, ND;

k_f = correction factor for CR₀ for concrete strength, ND;

k(PT) = correction factor for creep for precompression from post-tensioning, ND;

k_{v/s} = correction factor for base shrinkage due to volume to surface ratio (V/S), ND;

L = total length of a member, ft, m;

ND = indicates Non-Dimensional parameter;

P = prestressing force; lb, kN;

S = exposed surface area of a typical unit length of concrete member, in², mm²;

SH = contribution of shrinkage strain to shortening (Non-Dimensional ND);

s = a coefficient which depends on the type of cement, ND;

t = age of concrete in days;

T = change in temperature, °C, °F;
TEM = strain due to change in temperature;

V = volume of a typical unit length of concrete member, in\(^3\), mm\(^3\);

W = unit weight of concrete, pcf, kg/m\(^3\);

\(\alpha\) = coefficient of thermal expansion. / C\(^O\), /F\(^O\);

7 - REFERENCES


